SAFE HANDS & IIT-ian's PACE

LEAP TEST# 04 (JEE) ANS KEY Dt. 08-12-2023

PHYSICS		CHEMISTRY		MATHS	
Q. NO.	[ANS]	Q. NO.	[ANS]	Q. NO.	[ANS]
1	C	31	A	61	C
2	D	32	A	62	C
3	D	33	A	63	C
4	B	34	A	64	A
5	C	35	D	65	B
6	C	36	A	66	A
7	D	37	A	67	B
8	C	38	C	68	C
9	B	39	B	69	B
10	B	40	D	70	C
11	A	41	D	71	A
12	D	42	B	72	B
13	D	43	D	73	C
14	B	44	B	74	B
15	B	45	B	75	A
16	A	46	C	76	C
17	C	47	B	77	D
18	B	48	C	78	A
19	A	49	D	79	C
20	B	50	A	80	D
21	45	51	6	81	9
22	4	52	4	82	4
23	5	53	4	83	8
24	10	54	2	84	6
25	172	55	3	85	11

SAFE HANDS & PACE
LT-4 (JEE) Maths Solutions

: ANSWER KEY :

61)	c	62)	c	63)	c	64)	a	77)	d	78)	a	79)	c	80)	d
65)	b	66)	a	67)	b	68)	c	81)	9	82)	4	83)	8	84)	6
69)	b	70)	c	71)	a	72)	b	85)	11						
73)	c	74)	b	75)	a	76)	c								

: HINTS AND SOLUTIONS :

Single Correct Answer Type

61 **(c)**

$N_i = {}^5C_k \times {}^4C_{5-k}$

$N_1 = 5 \times 1$

$N_2 = 10 \times 4$

$N_3 = 10 \times 6$

$N_4 = 5 \times 4$

$N_5 = 1$

$N_1 + N_2 + N_3 + N_4 + N_5 = 126$

62 **(c)**

Example 22 page 121 NCERT Mathematics

63 **(c)**

The first digit a can take any one of 1 to 8

The third digit c can take any one of 0 to 9

When a =1, b can take any one of 2 to 9=8 values

When $a = 2$, b can take any one of 3 to 9=7 values

When $a = 3$, b can take any one of 4 to 9=6 values

… … … …

… … … …

When $a = 8, b$ can take any one $(b = 9) = 1$ values Thus, the number of total numbers

$$= (8 + 7 + 6 + \ldots + 2 + 1) \times 10 = \frac{8 \times 9}{2} \times 10 = 360$$

64 **(a)**

Required number of arrangements

$$= \frac{6!}{2!\,3!} - \frac{5!}{3!} = 60 - 20 = 40$$

65 **(b)**

A triangle is obtained by joining three non-collinear point

∴ The total number of triangles $= {}^{18}C_3 - {}^5C_3 = 806$

66 **(a)**

The number of ways of choosing a committee if there is no restriction is

$^{10}C_4 \cdot {}^9C_5 = \frac{10\,!}{4\,!\,6\,!} \cdot \frac{9\,!}{4\,!\,5\,!} = 26460$

The number of ways of choosing the committee if both Mr. A and Ms. B are included in the committee is $^9C_3 \cdot {}^8C_4 = 5880$

Therefore, the number of ways of choosing the committee when Mr. A and Ms. B are not together $= 26480 - 5880 = 20580$

67 **(b)**

Since, first the 2 women select the chairs amongst 1 to 4 in $^4P_2$ ways.

Now, from the remaining 6 chairs three men could be arranged in $^6P_3$ ways.

∴ Total number of arrangements= $^4P_2 \times {}^6P_3$

68 **(c)**

Given, $a_n = \sum_{r=0}^{n} \frac{1}{^nC_r}$

Let $b_n = \sum_{r=0}^{n} \frac{r}{^nC_r}$

Then, $b_n = \frac{0}{^nC_0} + \frac{1}{^nC_1} + \frac{2}{^nC_2} + \ldots\ldots + \frac{n}{^nC_n}$ ...(i)

$\Rightarrow b_n = \frac{n}{^nC_0} + \frac{n-1}{^nC_1} + \frac{n-2}{^nC_2} + \ldots\ldots + \frac{0}{^nC_n}$ ...(ii)

$[\because\ {}^nC_0 = {}^nC_n, {}^nC_1 = {}^nC_{n-1} \ldots.\text{as}\ {}^nC_r = {}^nC_{n-r}]$

On adding Eqs. (i) and (ii), we get

$$2b_n = \frac{n}{^nC_0} + \frac{n}{^nC_1} + \ldots\ldots + \frac{n}{^nC_n}$$

$$= n\left[\frac{1}{^nC_0} + \frac{1}{^nC_1} + \frac{1}{^nC_2} + \ldots\ldots + \frac{1}{^nC_n}\right]$$

$$\Rightarrow 2b_n = na_n$$

$$\therefore\ b_n = \frac{1}{2}na_n$$

69 **(b)**

Required number of ways = $^9C_4 = 126$

70 **(c)**

Example 20 page 119 NCERT Text book

71 **(a)**

We have,

$$\frac{^{2n+1}P_{n-1}}{^{2n-1}P_n} = \frac{3}{5}$$

$$\Rightarrow 5 \cdot {}^{2n+1}P_{n-1} = 3 \cdot {}^{2n-1}P_n$$

$$\Rightarrow 5 \cdot \frac{(2n+1)\,!}{(n+2)\,!} = \frac{3(2n-1)\,!}{(n-1)\,!}$$

$\Rightarrow \dfrac{5(2\,n+1)(2\,n)(2\,n-1)!}{(n+2)(n+1)n\,(n-1)\,!} = \dfrac{3\cdot(2n-1)\,!}{(n-1)\,!}$

$\Rightarrow 10(2\,n+1) = 3(n+2)\,(n+1)$

$\Rightarrow 3\,n^2 - 11\,n - 4 = 0 \Rightarrow n = 4$

72 **(b)**

We have the following ways of selections:

$p-identical\ things$	$q-identical\ things$	$Number\ of\ ways$
p	$r-p$	1
$p-1$	$r-(p-1)$	1
:	:	:
:	:	:
$r-q$	q	1

$\therefore$ Total number of ways $= p-(r-q)+1$ or, $q-(r-p)+1$

$= p+q-r+1$

73 **(c)**

From the first set, the number of ways of selection two lines $={}^4C_2$

From the second set, the number of ways of selection two lines $={}^3C_2$

Since, these sets are intersect, therefore they from a parallelogram,

$\therefore$ Required number of ways $= {}^4C_2 \times {}^3C_2$

$= 4 \times 3 = 12$

74 **(b)**

Let there be n participants. Then, we have

${}^nC_2 = 45$

$\Rightarrow \dfrac{n(n-1)}{2} = 45 \Rightarrow n^2 - n - 9 = 0 \Rightarrow n = 10$

75 **(a)**

Given, ${}^{2n+1}P_{n-1} : {}^{2n-1}P_n = 3:5$

$\Rightarrow \dfrac{(2n+1)!}{(n+2)!} \times \dfrac{(n-1)!}{(2n-1)!} = \dfrac{3}{5}$

$\Rightarrow \dfrac{(2n+1)2n}{(n+2)(n+1)n} = \dfrac{3}{5}$

$\Rightarrow 10\,(2n+1) = 3(n^2+3n+2)$

$\Rightarrow 3n^2 - 11n - 4 = 0$

$\Rightarrow (3n+1)(n-4) = 0$

$\Rightarrow\ \ n = 4 \qquad \left(n \neq -\dfrac{1}{3}\right)$

76 **(c)**

In a nine digits number, there are four even places for the four odd digits 3, 3, 5, 5

$\therefore$ Required number of ways $= \frac{4!}{2!2!} . \frac{5!}{2!3!} = 60$

77 **(d)**

Required number of ways

= Total number of ways in which 8 boys can sit

− Number of ways in which two brothers sit together

$= 8! - 7! \times 2! = 7! \times 6 = 30240$

78 **(a)**

A number between 5000 and 10,000 can have any of the digits 5,6,7,8,9 at thousand's place. So, thousand's place can be filled in 5 ways. Remaining 3 places can be filled by the remaining 8 digits in ${}^8P_3$ ways

Hence, required number $= 5 \times {}^8P_3$

79 **(c)**

The required number of ways

$= ({}^2C_1 \times {}^4C_2 + {}^2C_2 \times {}^4C_1) \times 3!$

$= (2 \times 6 + 1 \times 4)6 = 96$

80 **(d)**

In the word RAHUL the letters are (A, H, L, R, U)

Number of words starting with A= 4! = 24

Number of words starting with H=4! = 24

Number of words starting with L= 4! = 24

In the starting with R first one is RAHLU and next one is RAHUL.

$\therefore$ Rank of the word RAHUL= $3(24) + 2 = 74$

Integer Answer Type

81 **(9)**

x	x	

When two consecutive digits are 11, 22, etc $= 9 \cdot 9 = 81$

	0	0

When two consecutive digits are 0 0 = 9

	x	x

When two consecutive digits are $11, 22, 33, \dots = 9 \cdot 8 = 72$

Total number of number are $N = 162$

82 **(4)**

Number of arrangements are $2n!\, n!$

Given that $2n!\, n! = 1152$

$\Rightarrow (n!)^2 = 576$

$\Rightarrow n! = 24$

$\Rightarrow n = 4$

83 **(8)**

Let $n(A)$ = number of divisible by 60 $= (60, 120, \dots, 960) = 16$

$n(B)$ =number divisible by 24 $= (24, 48, \dots, 984) = 41$

$n(A \cap B)$ =number divisible by both

$= 120 + 240 + \cdots + 960 = 8$

Hence $n(A \cap B) = n(A) - n(A \cap B) = 16 - 8 = 8$

84 **(6)**

Number of numbers beginning with 1=120

1					

Number of number beginning with 2 =120

2					

Starting with 31……………….=24

3	1				

Starting with 3214……………….=2

3	2	1	4		

Finally =1

3	2	1	5	4	6

Hence unit place digit of 267^{th} number is 6

85 **(11)**

The number of divisors of 7875

$= (2 + 1)(3 + 1)(1 + 1) = 24$

This number includes the divisors 1 and 7875.

Remaining 22 divisors can be paired in 11 ways.

$\Rightarrow$ The required number of pairs $= 11$